F Test

(Testing Two Population Standard Deviations)

What is **F Test**?

It is a method used whenever we test two population standard deviations.

How do we set up H_0 and H_1 for this method?

Testing Type	H_0	H_1
Two-Tail Test	$\sigma_1 = \sigma_2$	$\sigma_1 \neq \sigma_2$
Left-Tail Test	$\sigma_1 \geq \sigma_2$	$\sigma_1 < \sigma_2$
Right-Tail Test	$\sigma_1 \leq \sigma_2$	$\sigma_1 > \sigma_2$

What information do we need to perform **F Test**?

We need sample size and sample standard deviation from each sample randomly selected from each population. It is recommended to organize the information in the form of a table as shown below.

Sample 1	Sample 2
n_1	n_2
s_1	s_2

Where

- $s_1 > s_2$,This is a must. If it is not, simply switch the samples.
- **CTS** $F = \frac{s_1^2}{s_2^2}$, Always round to 3-decimal places when needed.
- **Ndf** $= n_1 - 1$ & **Ddf** $= n_2 - 1$.

Example:

Use the chart below to test the claim that two population standard deviations are the same.

Sample 1	Sample 2
$n_1 = 10$	$n_2 = 12$
$s_1 = 8$	$s_2 = 4$

Solution:

We begin by setting up H_0 and H_1.

$H_0 : \sigma_1 = \sigma_2$ Claim

$H_1 : \sigma_1 \neq \sigma_2$ TTT

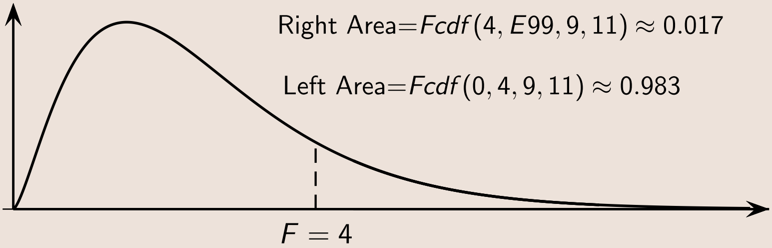
No significance level given, we must use $\alpha = 0.05$

Solution Continued:

We now find the following

- $s_1 > s_2 \checkmark$
- **CTS** $F = \dfrac{s_1^2}{s_2^2} = \dfrac{8^2}{4^2} = 4.$
- **Ndf** $= n_1 - 1 = 10 - 1 = 9.$
- **Ddf** $= n_2 - 1 = 12 - 1 = 11.$

We now draw the F distribution curve and clearly label.



Solution Continued:

Since it is a **TTT**,

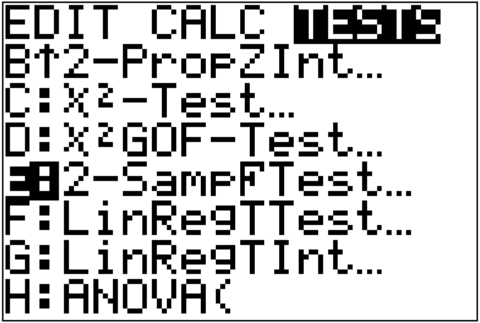
$$P - Value = 2 \cdot \boxed{\textbf{Smaller Area}} \approx 2 \cdot 0.017 \approx 0.034.$$

$p-$**value** is less than the significance level 0.05, therefore H_0 is invalid, and we reject the claim.

If we choose α to be 0.03, 0.02, or 0.01, then $p-$**value** is greater than the new significance level, therefore H_0 will be valid, and we fail to reject the claim.

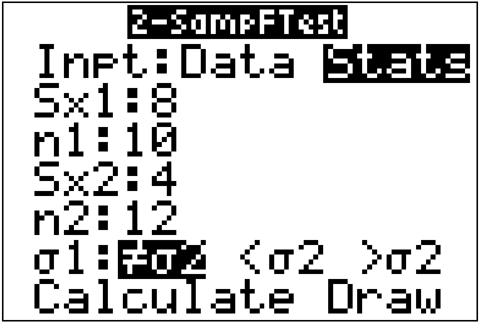
F Test & TI

Press **STAT**, go to **TESTS**, arrow down to find **2-SampFTest**.



F Test & TI

Now enter the data from the table in **2-SampFTest**, then press **Calculate** to execute **2-SampFTest**.



F Test & TI

The display below is from executing **2-SampFTest**.

```
2-SampFTest
 σ1≠σ2
 F=4
 p=.0342341936
 Sx1=8
 Sx2=4
↓n1=10
```

These results confirm our earlier findings.

Example:

A sample of 15 female students were randomly selected and the standard deviation of their ages was 9 years. In another sample of 10 male students, the standard deviation of their ages was 5 years.

Use 0.02 level of significance to test the claim that standard deviation of ages of all female students is different from standard deviation of ages of all male students.

Solution:

We begin by setting up H_0 and H_1.

$H_0 : \sigma_1 = \sigma_2$

$H_1 : \sigma_1 \neq \sigma_2$ Claim & TTT

Significance level given, $\alpha = 0.02$

Solution Continued:

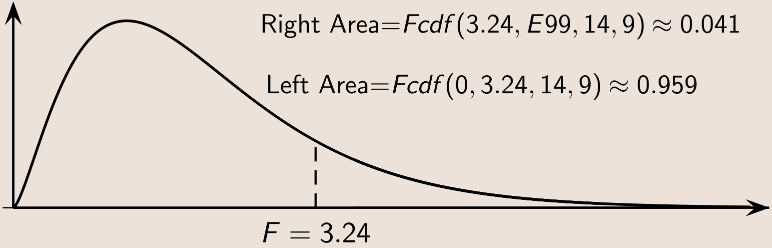
We can organize the given information in the following table.

Females	Males
$n_1 = 15$	$n_2 = 10$
$s_1 = 9$	$s_2 = 5$

- $s_1 > s_2 \checkmark$
- **CTS** $F = \frac{s_1^2}{s_2^2} = \frac{9^2}{5^2} = 3.24.$
- **Ndf** $= n_1 - 1 = 15 - 1 = 14.$
- **Ddf** $= n_2 - 1 = 10 - 1 = 9.$

Solution Continued:

We now draw the F distribution curve and clearly label.



Since it is a **TTT**,

$P - Value = 2 \cdot$ **Smaller Area** $\approx 2 \cdot 0.041 \approx 0.082$.

p−**value** is greater than the significance level 0.02, therefore H_1 is invalid, and we reject the claim.

Solution Continued:

We can verify these answers by using the **TI** command **2-SampFTest** to confirm their accuracy.

If we choose α to be 0.09 or 0.1, then $p-$**value** would be less than the new significance level, therefore H_1 will be valid, and we fail to reject the claim.

Example:

Use the chart below to suggest the significance level α to fail to reject the claim that $\sigma_1 > \sigma_2$.

Sample 1	Sample 2
$n_1 = 5$	$n_2 = 15$
$s_1 = 13$	$s_2 = 6$

Solution:

We begin by setting up H_0 and H_1.

$H_0 : \sigma_1 \leq \sigma_2$

$H_1 : \sigma_1 > \sigma_2$ Claim & RTT

Now we use the **TI** command **2-SampFTest** to find the **p-Value**.

p-Value $p = .013$

For H_1 to be valid, we want $p \leq \alpha$, so we want $0.013 \leq \alpha$.

Choose α to be 0.02, 0.03, 0.04, and so on.